Adaptive Control of Grid Quality for Computational Fluid Dynamics

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A grid adaptation technique is presented that improves the quality of computational grids for fluid flow calculations. The method adapts a grid directly to certain grid properties, such as grid skewness, grid stretching, or gridline kinking. A numerical mapping is employed that transforms the grid into a parametric space. Adaptive grid control is achieved by modifying the mapping functions through the effects of grid control sources extracted from the distribution of specific grid properties. A rediscretization of the modified parametric domain results in a grid-quality-adapted grid when mapped back to the physical space. The procedure retains the basic properties of the initial grid while reducing undesirable grid characteristics. The method is demonstrated with several examples.

Introduction

RECENT advances in computational fluid dynamics have created an unprecedented opportunity for the analysis of flow problems with increasingly complex physics and geometries. For instance, Navier-Stokes solutions are being sought for flowfields that may include strong shock waves, flow separation, mixing, and nonequilibrium molecular transports. Obtaining useful numerical solutions to these problems is complicated by the use of discrete grids. It is well known that different grids can yield different flow solutions for the same problem and the same flow solver because truncation errors depend on the grids used. Therefore, grids must be tailored to accommodate flow characteristics and geometrical features of the problems being solved. However, it becomes more difficult to generate suitable grids as physical and geometrical complexities increase.

In an attempt to alleviate some grid-related problems, unstructured grids have been used.[1,2] Unstructured grids can easily conform to complex shapes and allow local refinement without changing the rest of the grid. However, unstructured grids require more complicated data handling logic, increasing computation and memory requirements. Their irregularity may also sacrifice certain computational efficiencies associated with vectorization, parallelization, and implicit algorithms. Structured grids avoid the overhead costs of unstructured grids, but they may introduce other grid quality issues such as grid skewness, cell aspect ratio, gridline kinking, and nonsmooth stretching. Multiblock grids can be used to conform better to complex geometries without producing severe grid skewness or distortion, but may introduce serious gridline kinking or abrupt grid stretching across the block boundaries.[3] Grid generation, therefore, involves negotiation and compromise between different grid properties.

Many adaptive techniques have been developed to tailor grids to the characteristics of geometry or flow solutions.[4-6] Most adaptive methods control local grid density because truncation errors are directly related to cell size. Grid density is also important in the resolution of flow discontinuities, and must be compatible with viscous-length scales in order to accurately resolve viscous physics. However, there are other grid properties that affect computational accuracy and economy as well. Alignment of the grid with flow characteristics, such as shock waves, contact surfaces, and dividing streamlines, is important for the resolution of discontinuities. Grid kinking and grid stretching affect the order of accuracy of spatial discretizations, and can strongly influence convergence rates. These various grid properties are not independent of one another, and an improvement in one aspect of grid quality may cause a sacrifice in another aspect.

One example is given in Fig. 1, which shows an application of a solution-adaptive grid generation technique for an inviscid two-dimensional supersonic channel flow with compression and expansion corners. The upstream Mach number is 2.0 for this flow. Figures 1a and 1b show the initial grid used for the flow calculation and the resulting pressure contours. A solution-adaptive technique is used in order to improve the resolution of the shocks. The adapted grid and resulting pressure contours are shown in Figs. 1c and 1d. The adaptation process increases grid density in the regions of the shocks, resulting in sharp resolution of the discontinuities. However, the adaptation produces severe grid stretching and skewness that seriously impair the convergence rate of the solver. This implies that this particular solution-adapted grid creates a tradeoff between solution accuracy and economy. Thus, a compromise between conflicting grid properties is sought.

Grid-Quality-Adaptive Grid Generation

The present approach is based on a numerical mapping between the grid in the physical space and a domain in a two-dimensional parametric space. Grid adaptation is achieved by including the effects of grid control sources in the mapping functions. The overall method was first developed for geometry-adaptive surface grid generation,[7] in which the source strengths were extracted from local surface properties such as slope and curvature distributions. It was later used for solution-adaptive grid generation[8] by defining the source strengths from properties of a solution surface instead of a physical surface. For the present grid-quality-adaptive case, the source strengths are extracted from the distribution of a grid-quality parameter over the initial grid.

The grid adaptation procedure is depicted in Fig. 2. In this example, the method is used to reduce the excessive skewness in the solution-adapted grid of Fig. 1c, since grid skewness is known to cause convergence problems for the flow solver. The process begins by obtaining a parametric representation of the initial grid, shown again in Fig. 2a, by normalizing the grid index coordinates (ξ,η), into a unit square. The result is a

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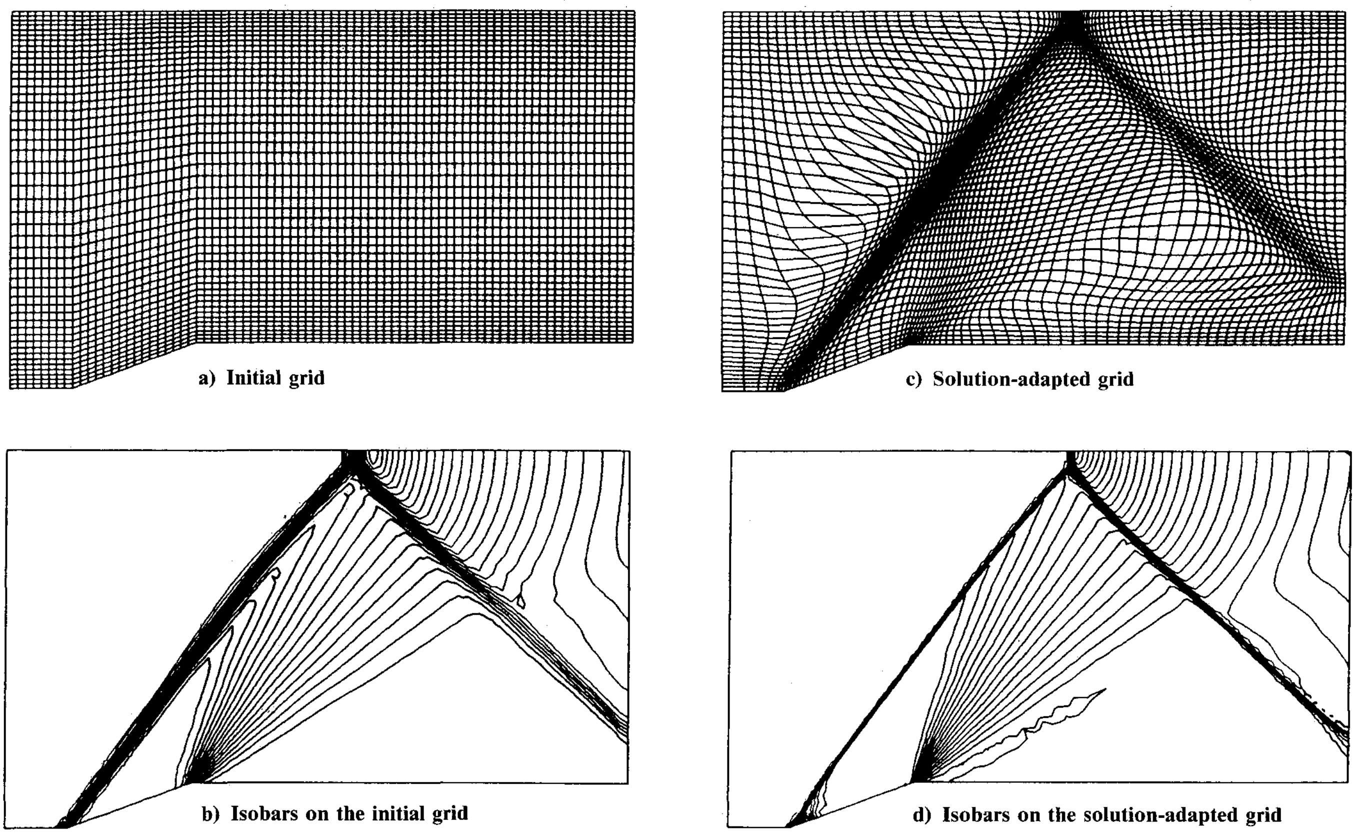


Fig. 1 Solution-adaptive grid generation for a supersonic channel flow.

uniformly discretized domain in parametric coordinates (s,t), as shown in Fig. 2b.

The next step is to define grid control sources at the centers of the cells in the parametric domain. Two sources are defined in each cell, σ_s and σ_t, to provide separate control in each parametric direction. The source strengths are determined so as to reflect local grid characteristics. A monitor function ϕ is chosen as a measure of some grid property. Then the source strengths can be defined as linear combinations of the monitor function and its derivatives in each parametric coordinate. For example,

$$\sigma_{kl}^s = w_0^s\phi + w_1^s\frac{\partial\phi}{\partial s} + w_2^s\frac{\partial^2\phi}{\partial s^2} \tag{1a}$$

$$\sigma_{kl}^t = w_0^t\phi + w_1^t\frac{\partial\phi}{\partial t} + w_2^t\frac{\partial^2\phi}{\partial t^2} \tag{1b}$$

where k and l are the indices of the cell containing the source. The w are input parameters allowing for different weights to be placed on the various derivatives of ϕ. Candidates for the monitor function, or grid quality parameter, include measures of grid skewness, grid kinking, grid stretching, cell aspect ratio, and cell volume. A combination of different grid quality parameters can also be used. Figures 2c and 2d show the source strength distributions for the two coordinate directions, using grid skewness as the grid quality parameter. The skewness for each cell is defined from the angle between the ξ and η directions. In this example, sources are defined from skewness itself, not from its derivatives.

Next, the mapping between the physical and parametric spaces is altered by including the effects of the grid control sources. This is done by defining a displacement at each grid point due to each source. The total displacement of each grid point is obtained by summing over all of the sources. This defines a modified set of parametric coordinates (s',t')

$$s_{ij}' = s_{ij} + \sum_{k,l} K_{ijkl}^s\sigma_{kl}^s \tag{2a}$$

$$t_{ij}' = t_{ij} + \sum_{k,l} K_{ijkl}^t\sigma_{kl}^t \tag{2b}$$

These coordinates are then renormalized into a unit square. That is, the values of s' for all points on a constant η line are normalized with the minimum and maximum values on that line, and likewise the values of t' for all points on a constant ξ line. Here, K_{ijkl}^s and K_{ijkl}^t are influence coefficients for the effects of a source (k,l) at a point (i,j). The coefficients are defined as exponentially decaying functions of the distance between the two points. For example,

$$K_{ijkl}^s = \frac{s_{ij}-s_{kl}}{d}\exp(-\alpha_s d) \tag{3a}$$

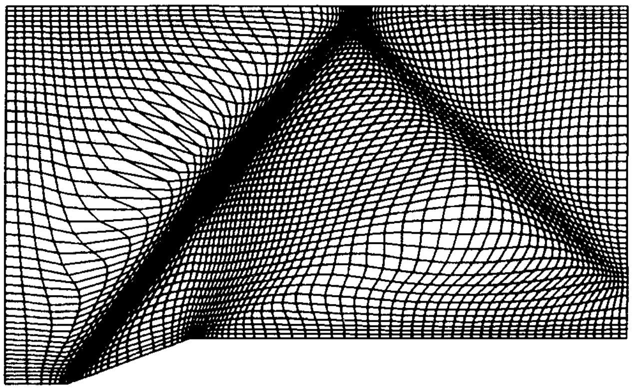
$$K_{ijkl}^t = \frac{t_{ij}-t_{kl}}{d}\exp(-\alpha_t d) \tag{3b}$$

where

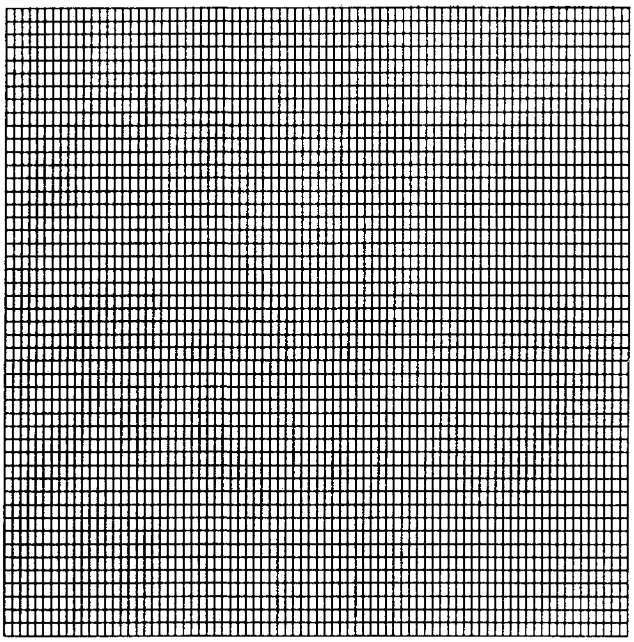
$$d = \sqrt{(s_{ij}-s_{kl})^2+(t_{ij}-t_{kl})^2}$$

and α_s and α_t are decay parameters for the two directions. The definition of the influence coefficients can vary, depending on the choice of monitor function and source strength definitions. The result of this second mapping is a modified parametric domain, shown in Fig. 2e, which is adapted to the distribution of the grid quality parameter. Gridlines are displaced by the source effects, distorting the parametric space in regions of strong sources.

The next step is to rediscretize the modified parametric domain. In this example, a uniform grid identical to that in Fig. 2b is used. These points provide the adapted grid when

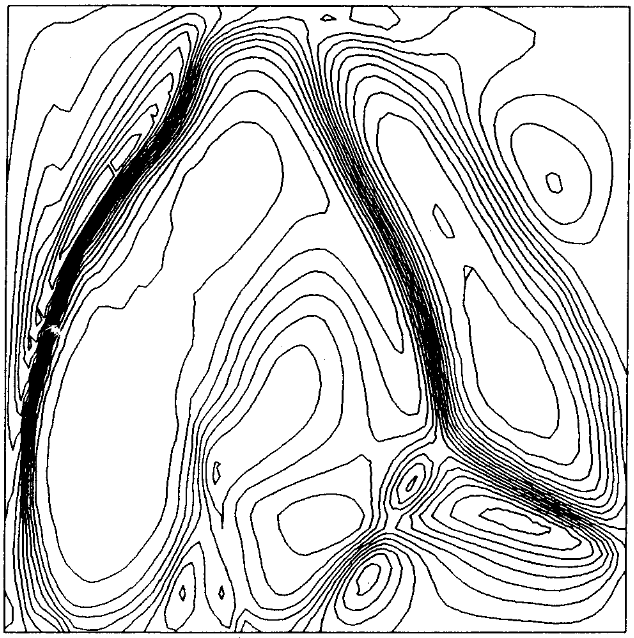


a) Initial grid

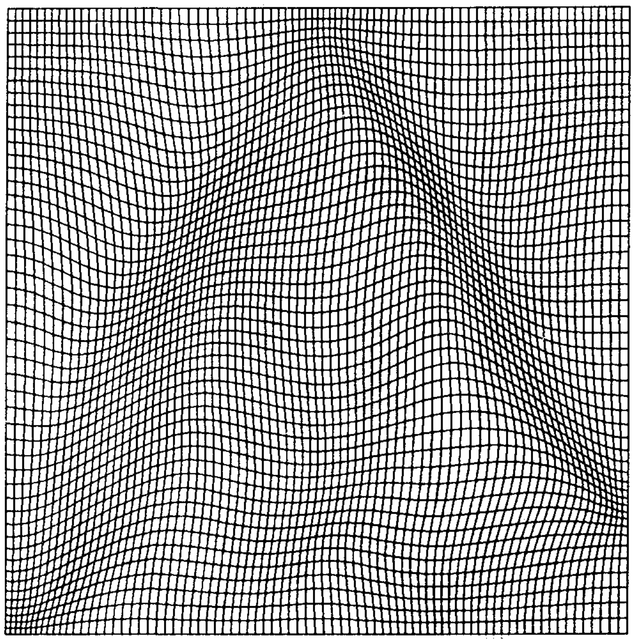


b) Parametric domain, (s,t)

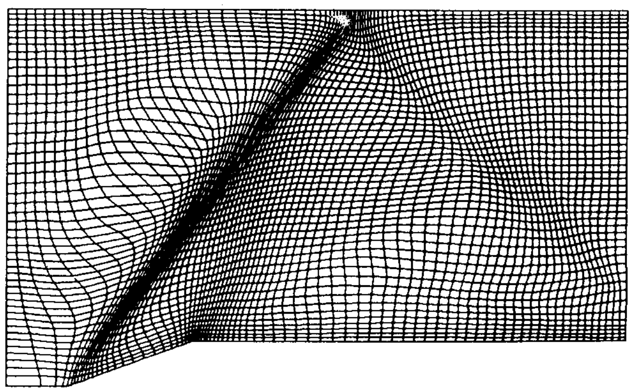
c) Source distribution, σ^s



d) Source distribution, σ^t



e) Modified parametric domain, (s',t')



f) Grid-quality-adapted grid

Fig. 2 Grid-quality-adaptive procedure for skewness reduction.

mapped back to the physical domain. This inverse mapping is performed by first locating the cell in the modified parametric domain that contains a new grid point, and then interpolating the physical coordinates of the point from the known values at the cell vertices. Figure 2f shows the resulting grid, which has been adapted to the distribution of the chosen grid-quality parameter.

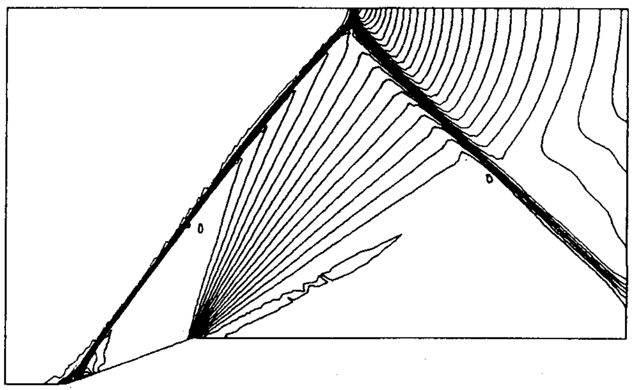
The combination of the weights in Eqs. (1) and the decay parameters in Eqs. (3) controls the overall degree of adaptation. The decay parameters determine the locality of the source influences. For given decay parameters, the values of weights are restricted to avoid a potential overlap in the modified parametric domain. For zero weights, the parametric domain is not modified and the initial grid is reproduced. Also, the effect of a source is insignificant over much of the domain, due to the decay with distance; therefore, the summations in Eqs. (2) can be performed over a partial domain defined by a cutoff radius. This significantly reduces the adaptation cost, and is very critical for three-dimensional applications.

The method provides many advantages stemming from the use of the parametric mappings and grid control sources. For instance, the basic characteristics of the initial grid can be retained while adapting to grid quality. The use of grid control sources allows for linear combinations of different controls based on the superposition principle. And the grid can be adapted to more than one monitor function through a series of mappings. The source formulation also promotes smooth variations in the grid, even with irregularly distributed sources. The adaptation process can be repeated in a cyclic manner if satisfactory results are not achieved after a single application.

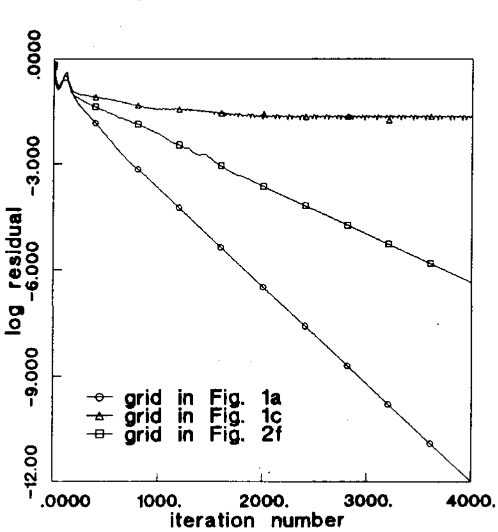
Results and Discussion

Several problems are presented to demonstrate different applications of the proposed grid adaptation technique. The first example concerns the channel flow of Figs. 1 and 2. As previously discussed, solution-adaptive grid techniques improve solution accuracy by controlling grid density, but increased grid skewness and stretching may impair the convergence. Figure 3a shows computed pressure contours for the grid of Fig. 2f, which was adapted to reduce grid skewness. The shock resolution of this solution compares very well with that of Fig. 1d. The convergence histories of solutions obtained on all three of the grids in Figs. 1a, 1c, and 2f are compared in Fig. 3b. All calculations were initialized with the freestream condition, so that the effects of grid properties alone could be seen. The solution on the initial grid converges very well, but the high skewness of the solution-adapted grid prevents proper convergence. After adapting for skewness, the solution is seen to converge, but at a slower rate than with the initial grid. These three grids and their corresponding solutions demonstrate a tradeoff between solution accuracy and convergence. Reduction of grid skewness using the proposed method produces a compromise between the initial grid and the solution-adapted grid.

The second example uses a grid about a blunt wedge, shown in Fig. 4a. The grid was generated in two separate blocks, resulting in the sharply kinked gridlines at the block boundary. The grid kinking causes problems for many flow solvers, resulting in large errors or severe stability limitations. For this case, grid kinking is used as the grid quality parameter. Monitor functions for control in the s and t directions are calculated from the turning angles of gridlines in the η and ξ direc-

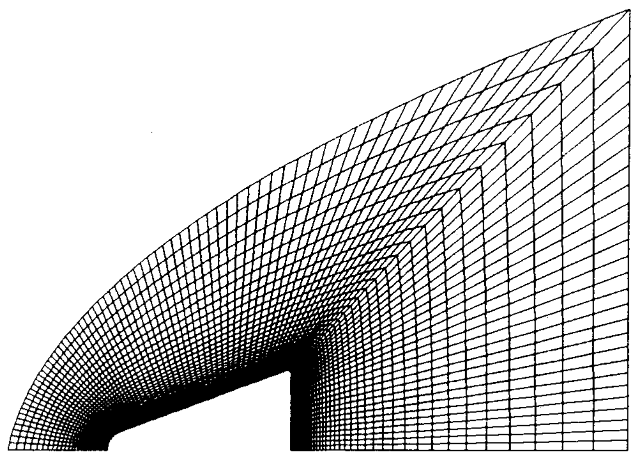


a) Isobars on the grid-quality-adapted grid in Fig. 2f

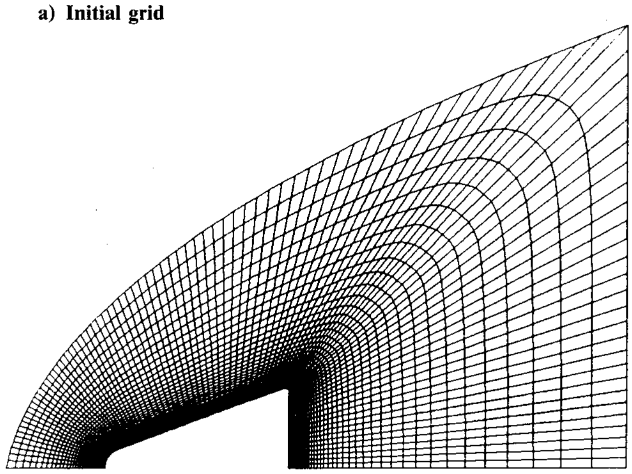


b) Comparison of convergence history

Fig. 3 Effects of grid skewness reduction on flow solution and convergence.

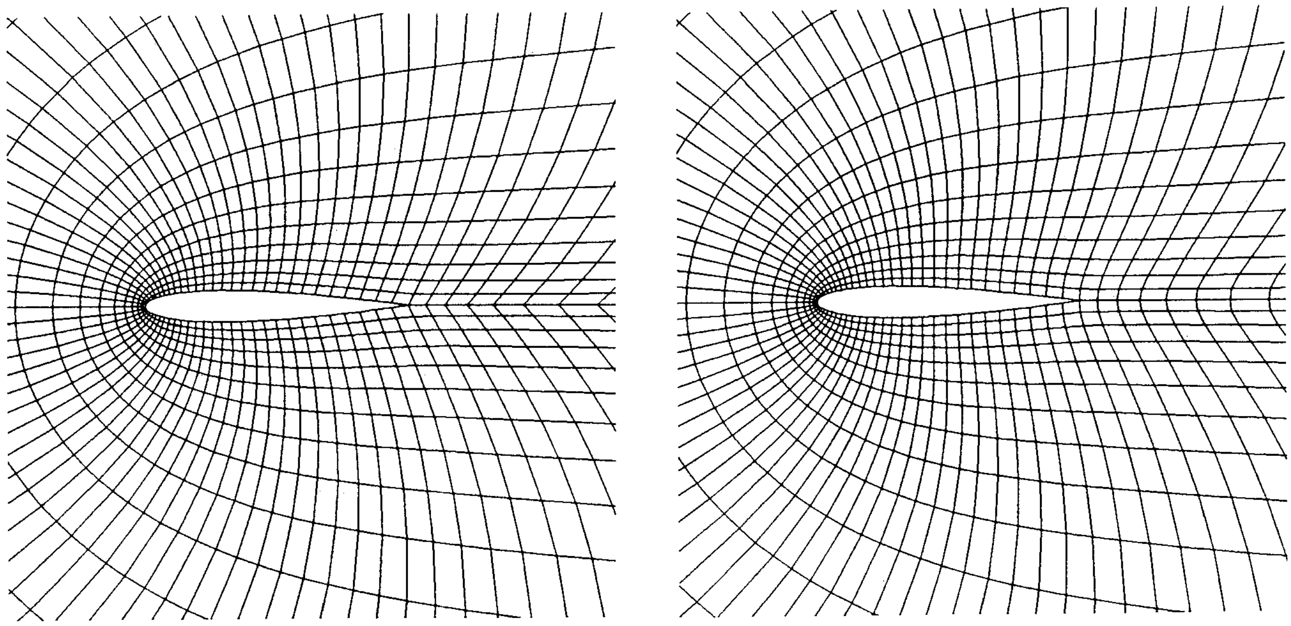


a) Initial grid



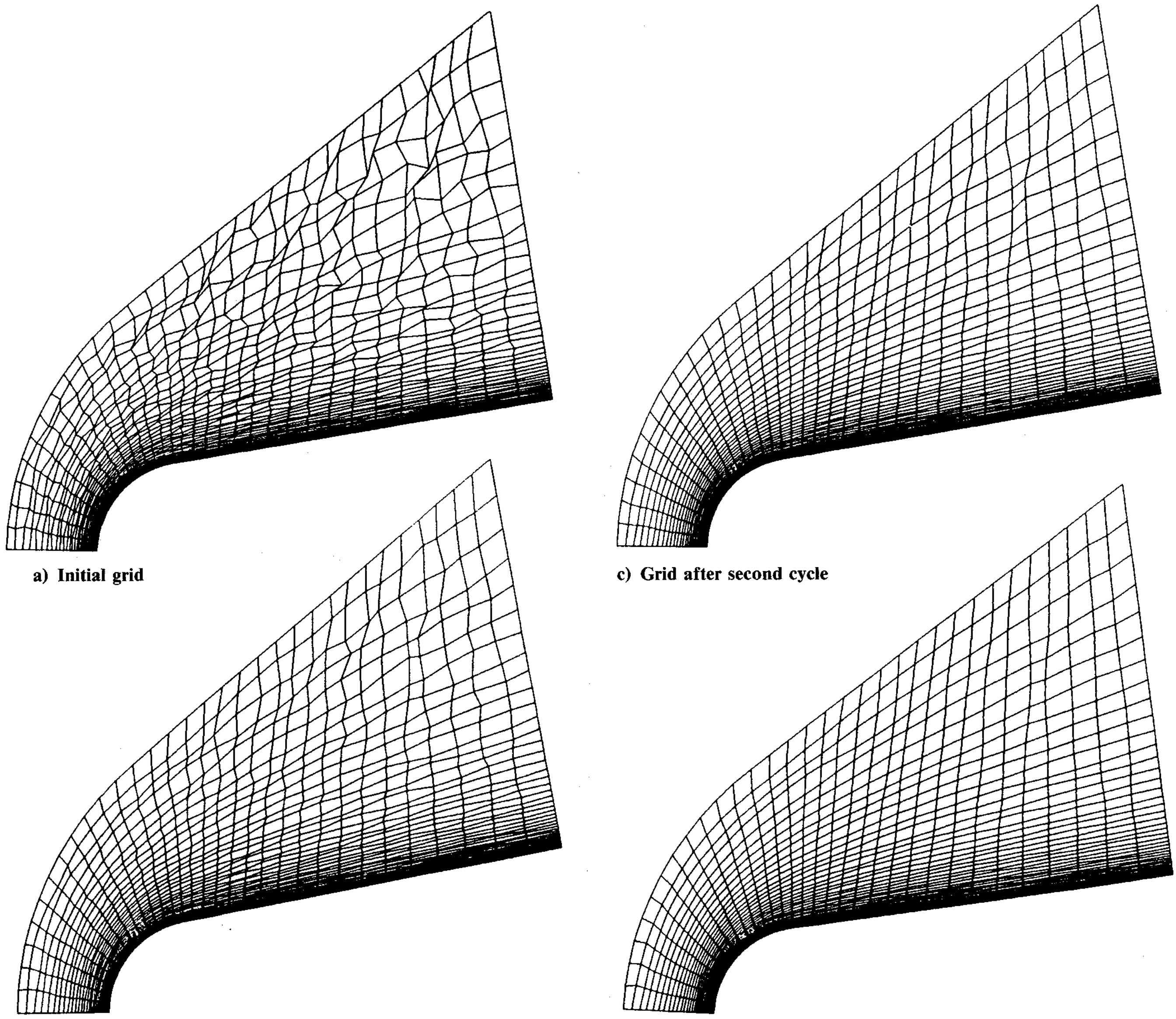
b) Adapted grid

Fig. 4 Reduction of grid kinking.



a) Initial grid b) Adapted grid

Fig. 5 Improvement of grid orthogonality at boundary.



a) Initial grid

c) Grid after second cycle

b) Grid after first cycle

d) Grid after third cycle

Fig. 6 Improvement of grid smoothness.

tions, respectively. The resulting adapted grid is shown in Fig. 4b. The adaptation procedure improves grid quality by reducing the kinks near the block boundary, but leaves the rest of the grid virtually unchanged.

Another demonstration of the technique is given in Fig. 5 for a *C*-mesh grid around a NACA 0012 airfoil. Boundary behavior of gridlines is especially important in viscous flow calculations for accurate predictions of skin friction and heat transfer rates. Therefore, orthogonal or nearly orthogonal grids are desired at configuration boundaries. Figure 5a shows the initial grid for the configuration. This grid is clearly too coarse for viscous flow calculations; it is used here simply to demonstrate the adaptation method. In this case, control sources are defined only in grid cells along the airfoil surface and wake line. Deviation from orthogonality of the radial gridlines with the body or wake surface is used for the monitor function. The adapted grid, shown in Fig. 5b, possesses good orthogonality near the surface and the wake, and blends smoothly into the rest of the grid.

The final example demonstrates successive applications of the method to improve the quality of a randomly scrambled grid for the nose section of a blunt wedge, shown in Fig. 6a. Here, grid kinking is again used as the grid quality parameter. The results of three successive applications of the method are shown in Figs. 6b–6d. The first adaptation produces a marked improvement in overall grid quality. After the third adaptation, most of the grid kinking has been removed. Further improvements could be made by adapting with a different grid-quality parameter, such as local stretching ratio. This example implies that the proposed technique can also be used as a grid generation tool, by repeated application with various grid-quality parameters, starting from an arbitrarily defined initial grid.

Concluding Remarks

This paper presents a method for improving grid quality using an adaptive technique to correct undesirable grid properties. Grids are modified by the influence of grid control sources extracted from the distribution of specific grid-quality parameters. The use of grid control sources allows for adaptation to different grid properties while preserving the basic characteristics of the initial grid. The method can be applied to grids obtained from any grid generation or adaptation techniques in order to control specific grid properties. The versatility and effectiveness of the method are demonstrated with several two-dimensional examples. An extension of the method to three dimensions is being investigated.

Acknowledgments

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